MASARYKOVA UNIVERZITA

Habilitation Thesis Reviewer's Report

Masaryk University	
Faculty	Faculty of Science
Procedure field	Theoretical Physics and Astrophysics
Applicant	Klaus Bering, PhD
Applicant's home unit, institution	Institute of Theoretical Physics and Astrophyics, Masaryk University
Habilitation thesis	*Odd scalar curvature in Batalin-Vilkovisky Geometry*
Reviewer	Dr Hovhannes Khudaverdian Senior Lecturer in Pure Mathematics
Reviewer's home unit, institution	The University of Manchester, UK

Bering' thesis "Odd scalar curvature in Batalin-Vilkovisky Geometry" is concerned with questions that are central for Batalin-Vilkovisky geometry.

The Batalin-Vilkovisky (BV) formalism was suggested in the seminal paper [1] in 1981. The initial aim of this formalism was to apply it to quantizing arbitrary singular Lagrangians in Quantum Field Theory. It has soon become clear that the BV formalism has much wider applications than its original use for degenerate Lagrangians.

Nowadays the BV formalism has become a powerful tool and source of ideas in such areas of mathematical and theoretical physics as quantisation of Poisson brackets, L_∞ algebras, etc. etc. Moreover, the ideas of this method penetrated even more distant areas of mathematics such as algebraic topology and became quite recently a source of constructing topological quantum field theories (see the works by Cattaneo--Reshethikhin--Mnev).

In the BV formalism, one introduces, for every field, an antifield of the opposite parity, coming in this way to an odd symplectic space of fields and antifields. The original action of the theory together with its symmetries define the so called master-action, a function on the space of fields and antifields which obeys the classical BV equation, while the exponent of the master-action obeys the quantum master-equation. We arrive at new differential geometry, which is the geometry of an odd symplectic superspace. In this geometry, appear constructions that have no analogues in the usual symplectic geometry. One of very important objects of this geometry is so called BV Delta-operator, which defines the quantum master-equation. It is a second order differential operator Δ_{dv} which is invariant with respect to odd symplectic transformations preserving the volume form dv. In work [2], the nature of this operator was revealed and it was shown that this operator is naturally defined by another canonical odd Laplace-type operator Δ_0 acting on semidensities. Unfortunately there were substantial difficulties to write down an explicit expression for the canonical operator in arbitrary (not Darboux) coordinates, and this problem has been waiting for a solution.

In work [3], P. Ševera constructed a spectral sequence such that the canonical Delta-operator Δ_0 was the second differential of this spectral sequence; however, this construction did not help to obtain an explicit formula for the operator Δ_0.

A great achievement of Klaus Bering was that he constructed an explicit formula for the Delta-operator Δ_0 in his work [4].

One of intriguing properties of the operator Δ_0 is that this operator assigns to an arbitrary volume form dv an odd scalar function

$$s(x) = \frac{\Delta_0\sqrt{dv}}{\sqrt{dv}}$$

In paper [5], based on the results of [4], Batalin and Bering investigated geometry of this scalar function. They showed that this scalar function is equal (up to a multiplicative factor) to the odd scalar curvature of a connection compatible with a volume form dv. This is deep geometrical statement, and it will play important role in the future studying of this geometry.

This habilitation thesis is essentially devoted to the explanation of these results and to generalisation of some of them to the case of an odd Poisson manifold.

These results of the thesis are fundamental and of great importance for theoretical physics.

Odd symplectic geometry as an area of mathematics was born together with the BV formalism. Nowadays it is connected with many different fields of mathematical research and I believe it has an exciting future. Results established by Klaus Bering will undoubtedly play important role in it.

I would like also to emphasize that Klaus Bering is considered in the community as an expert on questions related with a wide range of application of supermathematics in theoretical physics.

[1] A. Batalin and G. A. Vilkovisky. *Gauge algebra and quantization* Phys. Lett., 102B:27--31, 1981.

[2] H. M. Khudaverdian. *Semidensities on odd symplectic supermanifolds.* Comm. Math. Phys., 247(2):353--390, 2004.

[3] P. Ševera. *On the origin of the BV operator on odd symplectic supermanifolds.* Lett. Math. Phys. **78** (2006), no. 1, 55--59.

[4] K. Bering. *A note on semidensites in antisymplectic geometry.* J.Math.Phys. **47** 2006. (hep-th/0604)

[5] I. A. Batalin and K. Bering. *Odd scalar curvature in field-antifield formalism.* J.Math.Phys. 49:033515, (2008).

Conclusion

It is absolutely certain that the habilitation thesis submitted by Klaus Bering meets the high requirements applicable to habilitation theses in the field of Theoretical Physics and Astrophysics.

In Manchester on 26 August 2018