

HABILITATION THESIS REVIEWER'S REPORT

Masaryk University

Applicant

Phuoc-Tai Nguyen

Habilitation thesis

Boundary value problems for nonlinear elliptic equations with a Hardy potential

Reviewer

Prof.RNDr. Pavel Drábek, DrSc.

Reviewer's home unit, Institution

Department of Mathematics, Faculty of Applied Sciences, University of West Bohemia in Pilsen

The Habilitation Thesis of Phuoc-Tai Nguyen is devoted to the boundary value problems for nonlinear elliptic equations with a Hardy potential in a measure framework. Thesis consists of five chapters. The first one presents the state-of-the art in the field and addresses the main topics covered in the next four chapters. Concerning the whole thesis it has a unifying character. In particular, the author presents here results on the existence, nonexistence, uniqueness, a priori estimates and qualitative properties of solutions, a full characterization of isolated boundary singularities, removable singularities, etc. At the end of this chapter the reader can find exhaustive list of references counting 141 items.

Next four chapters represent four papers of the candidate which were published during the period 2017 – 2020 in renowned international journals: Journal of Differential Equations, Calculus of Variations in PDEs, Annales Inst. H. Poincaré and Advanced Nonlinear Studies. One of them is a single author paper, two of them are co-authored by Konstantinos Gkikas and one by Moshe Marcus, his postdoc mentor. In all cases the contribution of the candidate to the joint papers was proportional. Two “elder” papers which represent chapters 2 and 3 treat absorption nonlinear terms and source nonlinear terms. Striking distinctions regarding the existence and uniqueness between the absorption case and source case are pointed out. Chapter 4 is devoted to an extension of results in the previous chapters to more general equations and systems. The last chapter 5 focuses on the case where nonlinear terms depend on both solutions and their gradient. All above papers bring new and deep results which will be certainly frequently cited in future.

Besides above mentioned papers the candidate published since 2012 several other interesting results in other respected journals like Journal of Functional Analysis, Proc. London Math. Society, Advances in Differential Equations and Advances in Nonlinear Analysis. The coauthor of his most cited paper on MathSciNet is worldwide known expert in the theory of nonlinear differential equations, Laurent Veron, who was probably his PhD Thesis advisor. The second most cited work of Phuoc-Tai Nguyen is a paper which is actually chapter 2 of this thesis and it is coauthored by Moshe Marcus, well known expert on weighted problems and Hardy potential. This is to say that among the coauthors of the candidate one can find „big shots“ which give credit to the scientific level of his work.

Habilitation Thesis of Phuoc-Tai Nguyen fulfills the high standards of research in the field of nonlinear partial differential equations. These results will be certainly appreciated by other mathematicians and according to the reviewer's opinion will serve as a basis for further achievement. The candidate is already known as an expert in the field. According to his impressive scientific record since 2012 when he defended his PhD Thesis I am convinced that he will publish many new and interesting results also in future.

Reviewer's questions for the habilitation thesis defence (number of questions up to the reviewer)

1. What are, according to your opinion, the most challenging problems in the scope of the topic dealt in your Habilitation Thesis you want to concentrate on in future?
2. Which from the results for the semilinear problems are extendable to the case of quasilinear ones, represented for example by the p -Laplacian operator?

Conclusion

The Habilitation Thesis entitled "Boundary value problems for nonlinear elliptic equations with a Hardy potential" by Phuoc-Tai Nguyen fulfills requirements expected of a Habilitation Thesis in the field of Mathematics – Mathematical analysis.

Date: April 26, 2021

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